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Towards the light front variables for high energy production processes

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Abstract. Scale invariant presentation of inclusive spectra in terms of light front variables is proposed. The variables introduced go over to the well-known scaling variables $x_F = 2p_z/\sqrt{s}$ and $x_T = 2p_T/\sqrt{s}$ in the high p_z and high P_T limits respectively. Some surface is found in the phase space of produced $\pi^{\pm}$-mesons in the inclusive reaction $\bar{p}p \to \pi^{\pm}X$ at 22.4 GeV/c, which separates two groups of particles with significantly different characteristics. In one of these regions a naive statistical model seems to be in a good agreement with data, whereas it fails in the second region.

The study of single particle inclusive processes [1] remains one of the simplest and effective tools for the investigation of multiple production of secondaries at high energies. The consequences of the limiting fragmentation hypothesis [2] and those of the parton model [3] and the principle of automodelity for strong interactions [4] have been formulated in this way.

An important role in establishing of many properties of multiple production is played by the choice of kinematic variables in terms of which observable quantities are presented (see in this connection, e.g. [5–7]). The variables which are commonly used are the following: the Feynman $x_F = 2p_z/\sqrt{s}$, rapidity $y = \frac{1}{2}\ln[(E+p_z)/(E-p_z)]$, transverse scaling variable $x_T = 2p_T/\sqrt{s}$ etc. In the case of azimuthal symmetry the surfaces of const x_F are the planes $p_z = x_F\sqrt{s}/2$, surfaces of constant y are the hyperboloids

$$p_z^2\left[\left(\frac{1+e^{2y}}{1-e^{2y}}\right)^2 - 1\right] - p_T^2 = m^2$$

and the surfaces of constant x_T are the straight lines $p_T = x_T\sqrt{s}/2$ in the phase space. Here we propose unified scale invariant variables for the presentation of single particle inclusive distributions, the properties of which are described below.

Consider an arbitrary 4-momentum $p_\mu(p_0, \vec{p})$ and introduce the light front combinations [8]:

$$p_{\pm} = p_0 \pm p_3 \qquad (1)$$

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If the 4-momentum p_μ is on the mass shell ($p^2 = m^2$), the combinations $p_{\pm}$, $\vec{p}_T$ (where $\vec{p}_T = (p_1, p_2)$) define the so called horospherical coordinate system (see, e.g. [9,10]) on the corresponding mass shell hyperboloid $p_0^2 - \vec{p}^2 = m^2$. Corresponding hyperboloid in the velocity space is the realization of the curved space with constant negative curvature, i.e. the Lobachevsky space.

Let us construct the scale and Lorentz (under the transformations of reference frames along the collision axis) invariant variables:

$$\xi^{\pm} = \pm\frac{p_{\pm}^c}{p_{\pm}^a + p_{\pm}^b} \qquad (2)$$

in terms of the 4-momenta p_μ^a, p_μ^b, p_μ^c of particles a, b, c, entering the inclusive reaction $a + b \to c + X$. Particles a and b can be hadrons, heavy ions, leptons. Note that the use of similar variables turned out to be successful in theoretical studies of relativistic composite systems (see, e.g. [11–25]), in theoretical and experimental studies of nuclear reactions with beams of relativistic nuclei (see, e.g. [22,26,27]) and in the study of quark confinement in QCD (see, e.g. [28]). Combinations like (1) appear also when considering the scale transformations [29] in the theory with fundamental length (see, e.g. [30]).

Invariant differential cross section in terms of $(\xi^{\pm}, \vec{p}_T^c)$-variables looks as follows

$$E^c\frac{d\sigma}{d\vec{p}^c} = \frac{|\xi^{\pm}|}{\pi}\frac{d\sigma}{d\xi^{\pm}dp_T^{c2}} \qquad (3)$$

It is interesting to note the properties of $\xi^{\pm}$-variables in some limiting cases. Let us choose the centre of mass frame, where:

$$\xi^{\pm} = \pm\frac{E^c \pm p_z^c}{\sqrt{s}} = \pm\frac{E^c + |p_z^c|}{\sqrt{s}} \; ; \qquad E^c = \sqrt{p_z^{c^2} + p_T^{c^2} + m^{c^2}} \tag{4}$$

The upper sign in (4) is used for the right hand side hemisphere and the lower sign for the left hand side hemisphere in the centre of mass frame.

Consider two limiting cases:

1) $|p_z^c| \gg p_T^c$-fragmentation region, according to the common terminology. In this case:

$$\xi^{\pm} \longrightarrow \frac{2p_z^c}{\sqrt{s}} = x_F \tag{5}$$

2) $p_T^c \gg |p_z^c|$-high p_T-region. In this case:

$$\xi^{\pm} \longrightarrow \frac{m_T^c}{\sqrt{s}} \longrightarrow \frac{p_T^c}{\sqrt{s}} = \frac{x_T}{2} \; ; \quad m_T^c = \sqrt{p_T^{c^2} + m^{c^2}} \tag{6}$$

Thus, in these two limiting regions $\xi^{\pm}$-variables go over to the well known variables x_F and x_T, which are intensively used in high energy physics. $\xi^{\pm}$-variables are related to x_F, x_T and rapidity y as follows:

$$\xi^{\pm} = \frac{1}{2}\left(x_F \pm \sqrt{x_F^2 + x_\perp^2}\right); \quad x_\perp = \frac{2m_T^c}{\sqrt{s}} \tag{7}$$

$$y = \pm\frac{1}{2}\ln\frac{(\xi^{\pm}\sqrt{s})^2}{m_T^{c^2}} \tag{8}$$

The region $|\xi^{\pm}| < m^c/\sqrt{s}$ is kinematically forbidden for the $\xi^{\pm}$-spectra integrated over all values ol $p_T^{c^2}$, and the region $|\xi^{\pm}| < m_T^c/\sqrt{s}$ is forbidden for the $\xi^{\pm}$-spectra at fixed values of $p_T^{c^2}$.

In the present paper we study the inclusive reaction $\bar{p}p \to \pi^{\pm}X$ at 22.4 GeV/c of the incident momentum. The details of the experiment can be found in [31]. In this case it is sufficient to study the right hand side hemisphere only, due to the CP-symmetry of the reaction.

In Fig. 1a the ξ^+-distribution of $\pi^{\pm}$-mesons is shown.

ξ^+-distribution has two features , which makes it differ from the corresponding x_F-distribution:

1) existence of the forbidden region near the point $\xi^{\pm} = 0$ (cross section vanishes in the region $|\xi^{\pm}| < m_\pi/\sqrt{s}$,
2) existence of maximum at some $\tilde{\xi}^{\pm}$ in the region of relatively small ξ^+.

It is convenient to introduce the variables:

$$\zeta^{\pm} = \mp\ln|\xi^{\pm}| \tag{9}$$

in order to enlarge the scale in the region of small $|\xi^{\pm}|$. The maximum at $\tilde{\zeta}^+$ is also observed in the invariant differential cross section $\frac{1}{\pi}\frac{d\sigma}{d\zeta^+}$. However, the region $\xi^+ > \tilde{\xi}^+$ goes over to the region $\zeta^+ < \tilde{\zeta}^+$ and vice versa (see Fig. 1b).

In order to study the nature of this maximum we have investigated the angular and p_T^2-distributions of $\pi^{\pm}$-mesons in the regions $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ and $\xi^+ > \tilde{\xi}^+$ $(\zeta^+ < \tilde{\zeta}^+)$ separately. The results are presented in Figs. 2a and b. The angular distribution of particles with $\xi^+ > \tilde{\xi}^+$ $(\zeta^+ < \tilde{\zeta}^+)$ is sharply anisotropic in contrast to the almost flat distribution of particles with $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$. The slopes of p_T^2-distributions differ substantially.

Note, that the surfaces of constant ξ^+ are the paraboloids

$$p_z^c = \frac{p_T^{c^2} + m^{c^2} - (\xi^{\pm}\sqrt{s})^2}{-2\xi^{\pm}\sqrt{s}} \tag{10}$$

in the phase space. Thus the paraboloid

$$p_z^c = \frac{p_T^{c^2} + m^{c^2} - (\tilde{\xi}^{+}\sqrt{s})^2}{-2\tilde{\xi}^{+}\sqrt{s}} \tag{11}$$

separates two groups of particles with significantly different characteristics.

It seems to be interesting to use $\xi^{\pm}$ and $\zeta^{\pm}$ variables in deep inelastic electro and weak production processes, in e^+e^--annihilation and in relativistic heavy ion collisions (see in this connection recent reviews [32–38] and references therein) and to perform also event by event analysis.

The almost flat $\cos\theta$-distribution in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ allows one to conclude that the thermal equilibrium seems to be reached. To describe the spectra in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ the simplest statistical model (see, e.g. [39]) with the Boltzman $f(E) \sim e^{-E/T}$ and the Bose-Einstein $f(E) \sim (e^{E/T} - 1)$ distributions has been used.

The distributions $\frac{1}{\pi}\frac{d\sigma}{d\zeta^+}$, $\frac{d\sigma}{dp_T^2}$ and $\frac{d\sigma}{d\cos\theta}$ look in this region as follows

$$\frac{1}{\pi}\frac{d\sigma}{d\zeta^+} \sim \int_0^{p_{T,max}^2} Ef(E)dp_T^2 \; , \tag{12}$$

$$\frac{d\sigma}{dp_T^2} \sim \int_0^{p_{z,max}} f(E)dp_Z \; , \tag{13}$$

$$\frac{d\sigma}{d\cos\theta} \sim \int_0^{p_{max}} f(E)p^2dp \; , \tag{14}$$

$$E = \sqrt{\vec{p}^{\,2} + m_\pi^2}, \; \vec{p}^{\,2} = p_z^2 + p_T^2 \tag{15}$$

where

$$p_{T,max}^2 = (\xi^+\sqrt{s})^2 - m_\pi^2 \tag{16}$$

$$p_{z,max} = \frac{p_T^2 + m_\pi^2 - (\tilde{\xi}^+\sqrt{s})^2}{-2\tilde{\xi} + \sqrt{s}} \tag{17}$$

$$p_{max} = \frac{-\tilde{\xi}^+\sqrt{s}\cos\theta + \sqrt{(\tilde{\xi}^+\sqrt{s})^2 - m_\pi^2\sin^2\theta}}{\sin^2\theta} \tag{18}$$

The experimental distributions $\frac{1}{\pi}\frac{d\sigma}{dp_T^2}$ and $\frac{d\sigma}{d\cos\theta}$ in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ have been fitted by (12), (13) and (14), respectively. The results of the fit given in Table 1 and Figs. 1b, 2a, 2b show satisfactory agreement with experiment. Thus the spectra of $\pi^{\pm}$-mesons in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ are satisfactorily described by

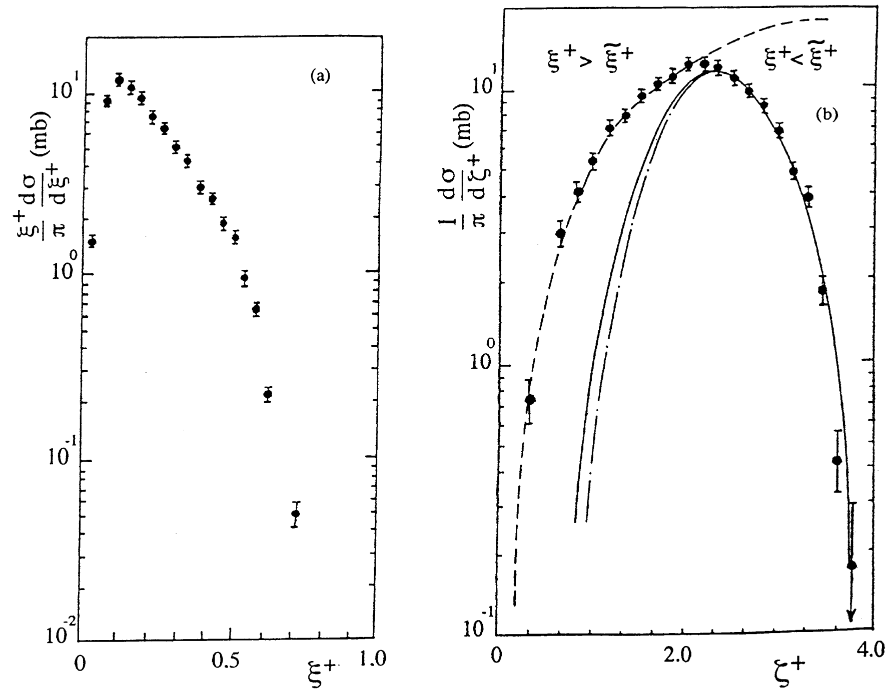

Fig. 1. a $\frac{\xi^+}{\pi}\ \frac{d\sigma}{d\xi^+}$-distribution of $\pi^\pm$ mesons in the reaction $\bar{p}p \to \pi^\pm X$ at 22.4 GeV/c, **b** $\frac{1}{\pi}\ \frac{d\sigma}{d\zeta^+}$-distribution of $\pi^\pm$ mesons in the reaction $\bar{p}p \to \pi^\pm X$ at 22.4 GeV/c, ——— fit of the data in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ by the Bose-Einstein distribution, —— · —— fit of the data in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ by the Boltzmann distribution, – – – fit of the data in the region $\xi^+ > \tilde{\xi}^+$ $(\zeta^+ < \tilde{\zeta}^+)$ by the formula $(1-\xi^+)^n$

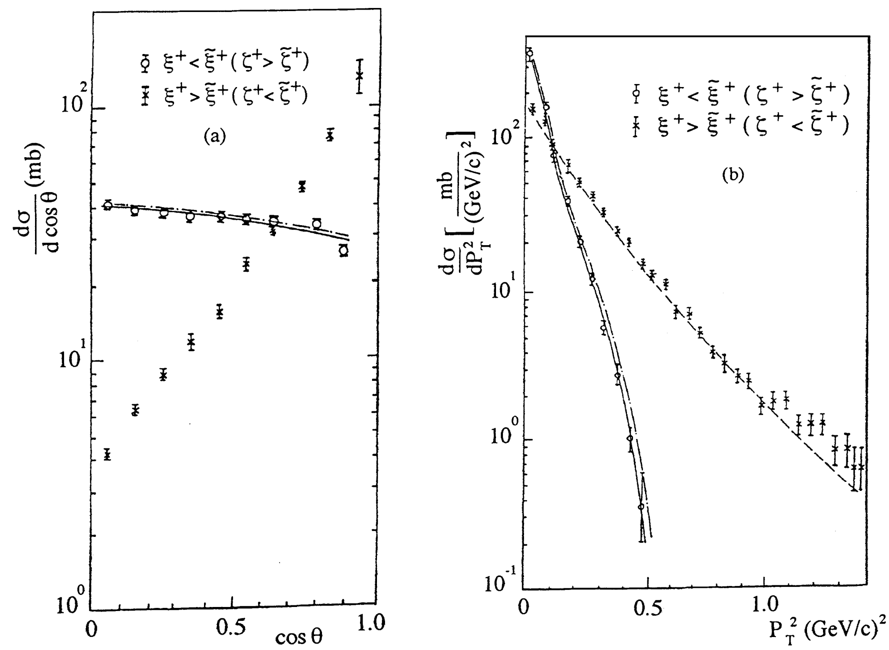

Fig. 2. a Angular distribution of $\pi^\pm$-mesons in the reaction $\bar{p}p \to \pi^\pm X$ at 22.4 GeV/c, ——— fit of the data in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ by the Bose-Einstein distribution, —— · —— fit of the data in the regoin $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ by the Boltzman distribution, **b** p_T^2-distribution of $\pi^\pm$ mesons in the reaction $\bar{p}p \to \pi^\pm X$ at 22.4 GeV/c, ——— fit of the data in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ by the Bose-Einstein distribution, —— · —— fit of the data in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ by the Boltzman distribution, – – – fit of the data in the region $\xi^+ > \tilde{\xi}^+$ $(\zeta^+ < \tilde{\zeta}^+)$ by the formula (20)

the formulae which follow from the statistical model. The same formulae when extrapolated to the region $\xi^+ > \tilde{\xi}^+$ $(\zeta^+ < \tilde{\zeta}^+)$ deviate from the data.

In the region $\xi^+ > \tilde{\xi}^+$ $(\zeta^+ < \tilde{\zeta}^+)$ ζ^+-distribution has been fitted by the formula:

$$\frac{1}{\pi}\frac{d\sigma}{d\zeta^+} \sim (1-\xi^+)^n = (1-e^{-\zeta^+})^n \qquad (19)$$

and the p_T^2-distribution by the formula:

$$\frac{d\sigma}{dp_T^2} \sim \alpha e^{-\beta_1 p_T^2} + (1-\alpha)e^{-\beta_2 p_T^2} \qquad (20)$$

Thus, the dependence $(1-\xi^+)^n$ is in a good agreement with data in the region $\xi^+ > \tilde{\xi}^+$ $(\zeta^+ < \tilde{\zeta}^+)$ and deviates from them in the region $\xi^+ < \tilde{\xi}^+$ $(\zeta^+ > \tilde{\zeta}^+)$ (see Fig. 1b).

Note that in the region $\xi^+ \to 1$ the parameterization (19) goes over to the well-known quark-parton model parameterization $(1-x)^n$ with $x = x_F = 2p_z/\sqrt{s}$. The results of the fit are given in Table 2 and Figs. 1b and 2b. Since the dependence $(1-x)^n$ which is derived for $x \to 1$ describes the data even in the region $x \to 0$ (where, in general, it must not work), but the dependence (19) deviates from the data in the region of small ξ^+, it seems that

Table 1. Results of the fits of $\frac{1}{\pi}\ \frac{d\sigma}{d\zeta^+}$, $\frac{d\sigma}{d\cos\theta}$ and $\frac{d\sigma}{dp_T^2}$-distributions in the region $\xi^+ < \tilde{\xi}^+$ ($\zeta^+ > \tilde{\zeta}^+$)

	T, GeV		$\chi^2/N_{D.F.}$	
	Bose-Einstein	Boltzman	Bose-Einstein	Boltzman
$\frac{1}{\pi}\ \frac{d\sigma}{d\zeta^+}$	0.134 ± 0.004	0.119 ± 0.003	10/8	12/8
$\frac{d\sigma}{d\cos\theta}$	0.091 ± 0.003	0.086 ± 0.003	16/7	15/7
$\frac{d\sigma}{dp_T^2}$	0.110 ± 0.001	0.105 ± 0.001	10/8	8/8

Table 2. Results of the fits of $\frac{d\sigma}{d\zeta^+}$, and $\frac{1}{\pi}\ \frac{d\sigma}{dp_T^2}$-distributions in the region $\xi^+ > \tilde{\xi}^+$ ($\zeta^+ < \tilde{\zeta}^+$)

	α	β_1 $(\mathrm{GeV/c})^{-2}$	β_2 $(\mathrm{GeV/c})^{-2}$	n	$\chi^2/N_{D.F.}$
$\frac{1}{\pi}\ \frac{d\sigma}{d\zeta^+}$	–	–	–	3.7 ± 0.1	7/9
$\frac{d\sigma}{dp_T^2}$	0.8 ± 0.03	6.0 ± 0.1	2.8 ± 0.3		45/29

the analysis of data in terms of $\xi^\pm$ and $\zeta^\pm$-distributions is more sensitive to the phenomenological models of multibody production at high energies than the analysis in terms of x_F.

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